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RF-4797

M. C. A. (Sem. - I) (A.T.K.T.) Examination

April / May – 2010

105 - Mathematical Foundation in Computer Science

Time : 3 Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. C. A. - 2**

Name of the Subject :
☛ **105 - MATHEMATICAL FOUNDATION IN COMPUTER SCIENCE**

☛ Subject Code No. : **4 7 9 7** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) Attempt all five questions.

(3) Questions numbered 1, 2, 3 carry 16 marks each.

(4) Questions numbered 4 and 5 carry 12 and 10 marks respectively.

1. Answer any **four** of the following:

(i) State multiplicative theorem of probability.
A problem in Statistics is given to three students A, B and C, whose probability of solving it are 1/3, 1/4 and 1/5 respectively. What is the probability that the problem will be solved.

(ii) The probability of a cricket match to be played between India and Sri Lanka at one of the three selected venues X, Y and Z are 0.25, 0.35 and 0.40 respectively. The probability that the match being disturbed at these three venues are 0.05, 0.04 and 0.02 respectively. A match is played and is disturbed, what is the probability that the match was played at X?

(iii) Define rank correlation.
The scores of 8 students in an examination in Mathematics and Statistics are given below:

Roll Nos.:	1	2	3	4	5	6	7	8
Marks in Mathematics :	70	48	58	55	54	50	60	52
Marks in Statistics :	62	47	53	60	55	68	51	48

Find rank correlation coefficient between them.

(iv) The age and blood pressures of 10 women are:

Age :	56	42	36	47	49	42	60	72	63	55
Blood Pressure :	147	125	118	128	145	140	155	164	149	150

Determine regression equation of blood pressure on age. Also estimate the blood pressure of a women whose age is 45 years.

(v) Probability that a pen manufactured by a company will be defective is 1/10. If 12 such pens are manufactured, find the probability that

(a) Exactly two pens will be defective,

(b) At least two pens will be defective.

(vi) Calculate mean and standard deviation for the following data:

Variable	Frequency	Variable	Frequency
0 -10	4	40-50	40
10 -20	10	50-60	30
20 -30	22	60-70	15
30 -40	32	70-80	8

2. Answer any **four** of the following:

(i) If $A = \begin{bmatrix} 4 & -5 & 1 \\ -5 & 3 & -3 \\ 1 & -3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} -8 & 7 & 2 \\ -7 & 9 & 5 \\ 3 & 5 & 13 \end{bmatrix}$

Compute (a) AB, (b) A^TB (c) AB^T (d) BA

(ii)(a) Prove that $\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (a^3-1)^2.$

(b) Prove that $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2.$

(iii) Obtain the value of the following determinant

$$\begin{vmatrix} 4 & 10 & 14 & 4 \\ 2 & 10 & 4 & 8 \\ 2 & 14 & -18 & 6 \\ 12 & 6 & 9 & 3 \end{vmatrix}.$$

(iv) For the matrix A given below show that $A^2 - 4A - 5I = 0$. Hence obtain A .

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

(v) For two square matrices $A = (a_{ij})_{nxn}$ and $B = (b_{ij})_{nxn}$, state the following results are true or false with appropriate reasons. Also verify with the following matrices:

(a) $Ax(A + B) = A^2 + AB$

(b) $(A + B)x(A - B) = A^2 - B^2$.

(vi) Solve the following system of equations using cramer's rule, if the solution exists.

$$2x + 4y + z = 3$$
$$3x + 2y - 2z = -2$$
$$x - y + z = 6.$$

3. Answer any **four** of the following:

(i) Define isomorphic graph. Prove that any two simple connected graphs with n vertices, all of degree two are isomorphic.

(ii) Define with appropriate example:
(a) k-regular graph, (b) Walk in a graph, (c) bipartite graph, (d) sub-graph.

(iii) Define with appropriate example:
(a) Simple graph, (b) finite and infinite graph, (c) complete graph, (d) isolated graph.

(iv) Describe Konigsberg bridge problem and its connection with Euler's graph.

(v) Define (a) Tree, and (b) level of a vertex in a binary tree.
Show that a graph is a tree if and only if it is minimally connected.

(vi) Show that the necessary and sufficient condition for a graph G to be a tree is that there is one and only one path between every pair of vertices in G.

4. Answer any **three** of the following:

(i) Describe utilities problem and discuss its relevance in graph theory.

(ii) Define degree of a vertex of a graph. Prove that the number of odd vertices in a graph is always even.

(iii) A computer while calculating the correlation coefficient between two variables X and Y, obtained the following constants:
$n=30,\ \sum X = 120,\ \sum Y = 90,\ \sum X^2 = 600,\ \sum Y^2 = 250,\ \sum XY = 356,$
It was discovered later at the time of checking that it had copied down two pairs of observations as:

X	Y
8	14
8	6

while the correct values were

X	Y
8	12
6	8

Compute the correct values of the correlation coefficient between X and Y.

(iv) If the distribution of incomes of a group of persons be assumed to be normal with mean 500 and the standard deviation Rs. 50. Estimate the proportion of individuals with income (a) below Rs. 550, and (b) between Rs. 550 and Rs. 650.

(v) If $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ then compute $(AB)^{-1}$.

5. Answer any **two** of the following:

(i) Let the probability density function of a random variable X be

$$f(x) = \frac{x}{2};\ 0 \le x \le 2.$$ Compute: (a) $P(x \le \frac{1}{2})$, (b) $P(x > \frac{3}{4})$,

(c) $P(\frac{1}{4} < x < \frac{3}{4})$, (d) E(x).

(ii) A salesman lives in a city A_1. He is supposed to visit the cities A_2, A_3, and A_4. The distance between these cities are as follows : (in Kms.)

$$A_1A_2 = 120,\ A_1A_3 = 140,\ A_1A_4 = 280,$$
$$A_2A_3 = 80,\ A_2A_4 = 180,\ A_3A_4 = 180,$$

Find the shortest round trip from A_1 through the other 3 cities.

(iii) $A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Then verify the following:

(a) A(B-C) = AB –AC, (b) $(A+B)x(A-B) = A^2 - B^2$.